2.1

2. Use set builder notation to give a description of each of these sets.

a) $\{0,3,6,9,12\}$

b) $\{-3,-2,-1,0,1,2,3\}$

c) $\{m,n,o,p\}$

7. Determine whether each of these pairs of sets are equal.

a) $\{1,3,3,3,5,5,5,5,5\}$, $\{5,3,1\}$

b) $\{\{1\}\},\{1,\{1\}\}$

c) $\emptyset,\{\emptyset\}$

11. Determine whether each of these statements is true or false.

a) $0 \in \emptyset$

b) $\emptyset \in \{0\}$

c) $\{0\} \subset \emptyset$

d) $\emptyset \subset \{0\}$

e) $\{0\} \in \{0\}$

f) $\{0\} \subset \{0\}$

g) $\{\emptyset\} \subseteq \{\emptyset\}$

19. Suppose that A, B, and C are sets such that $A \subseteq B$ and $B \subseteq C$. Show that $A \subseteq C$.

22. What is the cardinality of each of these sets?

a) $\emptyset$

b) $\{\emptyset\}$

c) $\{\emptyset,\{\emptyset\}\}$

d) $\{\emptyset,\{\emptyset\},\{\emptyset,\{\emptyset\}\}\}$

2.2

19. Show that if A, B, and C are sets, then $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$

a) by showing each side is a subset of the other side.

b) using a membership table.

21. Show that if A and B are sets, then

a) $A - B = A \cap \bar{B}$.

b) $(A \cap B) \cup (A \cap \bar{B}) = A$.

30. Draw the Venn diagrams for each of these combinations of the sets A, B, C and D.

a) $(A \cap B) \cup (C \cap D)$

b) $\bar{A} \cup \bar{B} \cup \bar{C} \cup \bar{D}$

c) $A - (B \cap C \cap D)$

53. Let $A_i = 1,2,3, \dots, i$ for $i = 1,2,3, \dots$. Find

a) $\bigcup_{i=1}^{n} A_i$.

b) $\bigcap_{i=1}^{n} A_i$.

2.3

2. Determine whether f is a function from Z to R if

a) $f(n) = \pm n$.

b) $f(n) = \sqrt{n^2 + 1}$.

c) $f(n) = 1/(n^2 - 4)$.

3. Determine whether f is a function from the set of all bit strings to the set of integers if

a) $f(S)$ is the position of a 0 bit in S.

b) $f(S)$ is the number of 1 bits in S.

c) $f(S)$ is the smallest integer i such that the i th bit of S is 1 and $f(S) = 0$ when S is the empty string, the string with no bits.

8. Find these values.

a) $\lfloor 1.1 \rfloor$

b) $\lceil 1.1 \rceil$

c) $\lfloor -0.1 \rfloor$

d) $\lceil -0.1 \rceil$

e) $\lceil 2.99 \rceil$

f) $\lceil -2.99 \rceil$

g) $\lfloor ½ + \lceil ½ \rceil \rfloor$

h) $\lceil \lfloor ½ \rfloor + \lceil ½ \rceil + ½ \rceil$

23. Determine whether each of these functions is a bijection from R to R.

a) $f(x) = 2x + 1$

b) $f(x) = x^2 + 1$

c) $f(x) = x^3$

d) $f(x) = (x^2 + 1)/(x^2 + 2)$

30. Let $S = -1,0,2,4,7$. Find $f(S)$ if

a) $f(x) = 1$.

b) $f(x) = 2x + 1$.

c) $f(x) = \lceil x/5 \rceil$.

d) $f(x) = \lfloor (x^2 + 1)/3$.

53. Show that if x is a real number and n is an integer, then

a) $x < n$ if and only if $\lfloor x \rfloor < n$.

b) $n < x$ if and only if $n < \lceil x \rceil$.

2.4

10. Find the first six terms of the sequence defined by each of these recurrence relations and initial conditions.

a) $a_n = -2a_{n-1}, a_0 = -1$

b) $a_n = a_{n-1} - a_{n-2}, a_0 = 2, a_1 = -1$

c) $a_n = 3a_{n-1}^2, a_0 = 1$

d) $a_n = na_{n-1} + a_{n-2}^2, a_0 = -1, a_1 = 0$

e) $a_n = a_{n-1} - a_{n-2} + a_{n-3}, a_0 = 1, a_1 = 1, a_2 = 2$

11. Let $a_n = 2^n + 5 \cdot 3^n$ for $n = 0,1,2, \ldots$.

a) Find a_0, a_1, a_2, a_3, and a_4.

b) Show that $a_2 = 5a_1 - 6a_0, a_3 = 5a_2 - 6a_1$, and $a_4 = 5a_3 - 6a_2$.

c) Show that $a_n = 5a_{n-1} - 6a_{n-2}$ for all integers n with $n \geq 2$.

12. Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if

a) $a_n = 0$.

b) $a_n = 1$.

c) $a_n = (-4)^n$.

d) $a_n = 2(-4)^n + 3$.

18. A person deposits $1000 in an account that yields 9% interest compounded annually.

a) Set up a recurrence relation for the amount in the account at the end of n years.

b) Find an explicit formula for the amount in the account at the end of n years.

c) How much money will the account contain after 100 years?

31. What is the value of each of these sums of terms of a geometric progression?

a) $\sum_{j=0}^{8} 3 \cdot 2^j$

b) $\sum_{j=1}^{8} 2^j$

c) $\sum_{j=2}^{8} (-3)^j$

d) $\sum_{j=0}^{8} 2 \cdot (-3)^j$

2.5

3. Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.
a) all bit strings not containing the bit 0
b) all positive rational numbers that cannot be written with denominators less than 4
c) the real numbers not containing 0 in their decimal representation
d) the real numbers containing only a finite number of 1 s in their decimal representation

4. Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.
a) integers not divisible by 3
b) integers divisible by 5 but not by 7
c) the real numbers with decimal representations consisting of all 1 s
d) the real numbers with decimal representations of all 1 s or 9 s

7. Suppose that Hilbert's Grand Hotel is fully occupied on the day the hotel expands to a second building which also contains a countably infinite number of rooms. Show that
the current guests can be spread out to fill every room of the two buildings of the hotel.

15. Show that if A and B are sets, A is uncountable, and $A \subseteq B$, then B is uncountable.

22. Suppose that A is a countable set. Show that the set B is also countable if there is an onto function f from A to B.

2.6

6. Find a matrix **A** such that

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 1 \\ 4 & 0 & 3 \end{bmatrix} \mathbf{A} = \begin{bmatrix} 7 & 1 & 3 \\ 1 & 0 & 3 \\ -1 & -3 & 7 \end{bmatrix}.$$

7. Let A be an $m \times n$ matrix and let 0 be the $m \times n$ matrix that has all entries equal to zero. Show that $A = 0 + A = A + 0$.

10. Let A be a 3×4 matrix, B be a 4×5 matrix, and C be a 4×4 matrix. Determine which of the following products are defined and find the size of those that are defined.

a) AB

b) BA

c) AC

d) CA

e) BC

f) CB

20. Let

$$\mathbf{A} = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}.$$

a) Find A^{-1}. [Hint: Use Exercise 19.]

b) Find A^3.

c) Find $(A^{-1})^3$.

d) Use your answers to (b) and (c) to show that $(A^{-1})^3$ is the inverse of A^3.

27. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

Find

a) $\mathbf{A} \vee \mathbf{B}$.

b) $\mathbf{A} \wedge \mathbf{B}$.

c) $\mathbf{A} \odot \mathbf{B}$.

31. In this exercise we show that the meet and join operations are commutative. Let A and B be $m \times n$ zero-one matrices. Show that

a) $A \vee B = B \vee A$.

b) $B \wedge A = A \wedge B$.

Chapter 2—Test 1

1. Let $A = \{a, c, e, h, k\}$, $B = \{a, b, d, e, h, i, k, l\}$, and $C = \{a, c, e, i, m\}$. Find each of the following sets.

(a) $A \cap B$

(b) $A \cap B \cap C$

(c) $A \cup C$

(d) $A \cup B \cup C$

(e) $A - B$

(f) $A - (B - C)$

2. Prove or disprove that if A, B, and C are sets then $A - (B \cap C) = (A - B) \cap (A - C)$.

3. Let $f(n) = 2n + 1$. Is f a one-to-one function from the set of integers to the set of integers? Is f an onto function from the set of integers to the set of integers? Explain the reasons behind your answers.

4. Suppose that f is the function from the set $\{a, b, c, d\}$ to itself with $f(a) = d$, $f(b) = a$, $f(c) = b$, $f(d) = c$. Find the inverse of f.

5. Find the values of $\sum_{j=1}^{100} 2$ and $\sum_{j=1}^{100} (-1)^j$.

6. Let $\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$. Find $\boldsymbol{AB}$ and $\boldsymbol{BA}$. Are they equal?

7. Let $\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Find the join, meet, and Boolean product of these two zero-one matrices.

Chapter 2—Test 2

1. Let A, B, and C be sets. Prove or disprove that $A-(B\cap C)=(A-B)\cup(A-C)$.

2. Consider the function $f(n)=2\lfloor n/2\rfloor$ from $\mathbf{Z}$ to $\mathbf{Z}$. Is this function one-to-one? Is this function onto? Justify your answers.

3. Show that the set of odd positive integers greater than 3 is countable.

4. Find $\displaystyle\sum_{j=1}^{100} 2j+5$ and $\displaystyle\sum_{j=5}^{100} 3^j$.

5. Prove or disprove that $\boldsymbol{AB}=\boldsymbol{BA}$ whenever $\boldsymbol{A}$ and $\boldsymbol{B}$ are 2×2 matrices.